UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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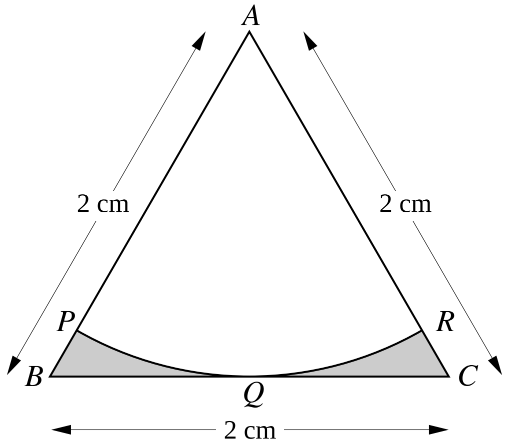
1 Solve the equation $\sin 2x = 2\cos 2x$, for $0° \leqslant x \leqslant 180°$. [4]

2 Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$. [4]

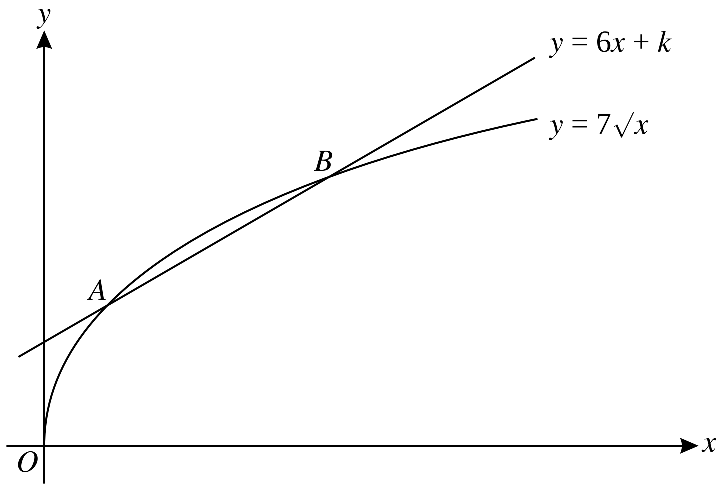
3



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

4 A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius, r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of k and the rate at which the mass is increasing on this day. [5]

5



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

6 Two vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.

(i) Find the values of p for which $\mathbf{u}$ is perpendicular to $\mathbf{v}$. [3]

(ii) For the case where $p = 1$, find the angle between the directions of $\mathbf{u}$ and $\mathbf{v}$. [4]

7 **(a)** The first two terms of an arithmetic progression are 1 and $\cos^2 x$ respectively. Show that the sum of the first ten terms can be expressed in the form $a - b\sin^2 x$, where a and b are constants to be found. [3]

(b) The first two terms of a geometric progression are 1 and $\frac{1}{3}\tan^2\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(i) Find the set of values of θ for which the progression is convergent. [2]

(ii) Find the exact value of the sum to infinity when $\theta = \frac{1}{6}\pi$. [2]

8 The function $\mathrm{f} : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geqslant p$, where k and p are constants.

(i) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]

(ii) State the range of f in terms of k. [1]

(iii) State the smallest value of p for which f is one-one. [1]

(iv) For the value of p found in part **(iii)**, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}, giving your answers in terms of k. [4]

9 The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$. The mid-point of AC is M and the perpendicular bisector of AC cuts the x-axis at B.

(i) Find the equation of MB and the coordinates of B. [5]

(ii) Show that AB is perpendicular to BC. [2]

(iii) Given that $ABCD$ is a square, find the coordinates of D and the length of AD. [2]

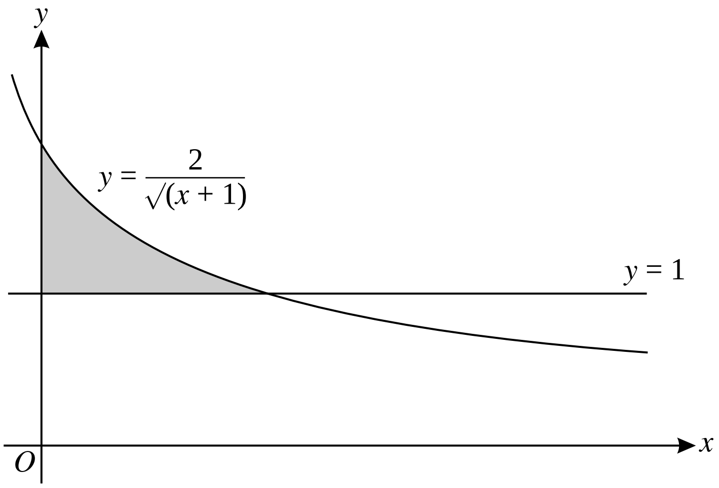
10 It is given that a curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - 2x^2 + x$.

(i) Find the set of values of x for which the gradient of the curve is less than 5. [4]

(ii) Find the values of $\mathrm{f}(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

[Question 11 is printed on the next page.]

11



The diagram shows the line $y = 1$ and part of the curve $y = \dfrac{2}{\sqrt{(x+1)}}$.

(i) Show that the equation $y = \dfrac{2}{\sqrt{(x+1)}}$ can be written in the form $x = \dfrac{4}{y^2} - 1$. [1]

(ii) Find $\displaystyle\int \left(\frac{4}{y^2} - 1\right) \mathrm{d}y$. Hence find the area of the shaded region. [5]

(iii) The shaded region is rotated through 360° about the ***y*-axis**. Find the exact value of the volume of revolution obtained. [5]